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Your Roll No. :

No. of Q. Paper : 610 I

Unique Paper Code : 32357504

Name of the Course : B.Sc.(Hons.)
Mathematics : DSE -I

Name of the Paper : Mathematical Finance

Semester : V

Time : 3 Hours Maximum Marks : 75

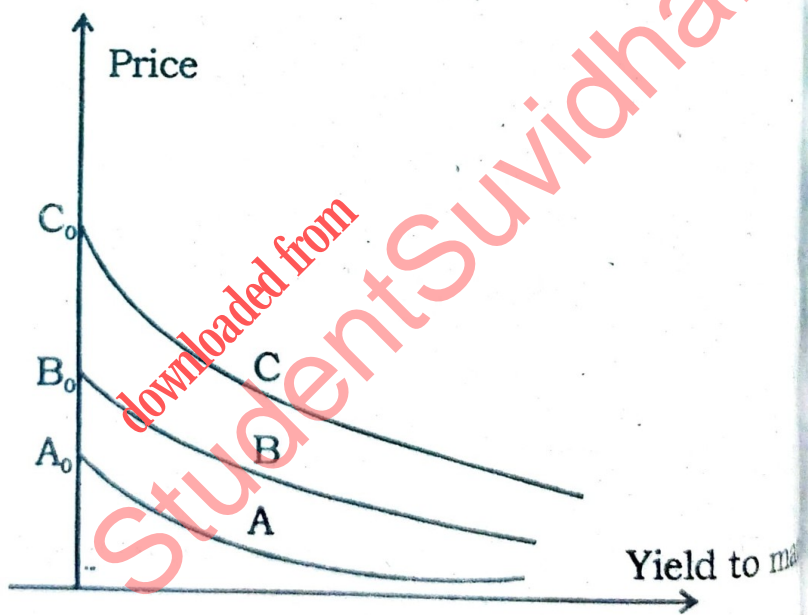
Instructions for Candidates :

- (a) Write your Roll No. on the top immediately on receipt of this question paper.
- (b) Attempt any **two** parts from each question.
- (c) Following values may be used if needed
 $e^{0.025} = 1.0253$, $e^{-0.025} = 0.975$, $e^{0.0125} = 1.0125$
and $e^{-0.0125} = 0.9875$.
- (a) State and prove annuity formula. 6
- (b) A young couple has made a non-refundable deposit of the first month's rent (equal to \$1,000) on a 6-month apartment lease. The next day they find a different apartment that they like just as well, but its monthly rent

P.T.O.

is only \$900. They plan to be in the apartment only 6 months. Should they switch to a new apartment? What if they plan to stay a year? Assume an interest rate of 12%

- (c) Consider three bonds, each with maturity 30 years having respective coupon rates 10%, 5%, 0%.



Match price-yield curves A, B, C with three given bonds. Find A_0 , B_0 and C_0 ; Price of intersection of price-yield curves A with the price axis. Justify your answer

(a) Find future value and present value of the cash flow stream $(-1, 2, 2)$, having each period as one year when the prevailing interest rate is 10% per annum. Also find IRR for the given cash flow stream. 6

(b) Suppose that you have the opportunity to plant trees that later can be sold for lumber. This project requires an initial outlay of money in order to purchase and plant the seedlings. No other cash flow occurs until the trees are harvested. However, you have a choice as to when to harvest: after 1 year or after 2 years. If you harvest after 1 year, you get your return quickly; but if you wait an additional year, the trees will have additional growth and the revenue generated from the sale of trees will be greater. Assume that the cash flow streams associated with these two alternatives are

(i) $(-1, 2)$ cut early

(ii) $(-1, 0, 3)$ cut later.

Also assume that the prevailing interest rate is 10%. Find out when is it best to cut the trees under NPV criteria. 3+3

(c) Define portfolio return and derive expression for variance of portfolio return

3. (a) What do the various symbols stand for in the following bond price formula?

$$P = \frac{F}{[1 + (\lambda/m)]^n} + \frac{C}{\lambda} \left\{ 1 - \frac{1}{[1 + (\lambda/m)]^n} \right\}$$

Should the price be higher or lower if the yield is higher?

(b) The correlation ρ between assets A and B is 0.1, and other data are given in the table

below where $\rho = \frac{\sigma_{AB}}{\sigma_A \sigma_B}$

Asset	\bar{r}	σ
A	10%	15%
B	18%	30%

(i) Find the proportions α of A and $(1-\alpha)$ of B that define a portfolio of A and B having minimum standard deviation

- (ii) What is the value of this minimum standard deviation ?
- (iii) What is the expected return of this portfolio ? 6.5
- (c) (i) Define spot rate s_t for t years. How is s_t determined under yearly, m periods per year and continuous compounding conventions ? 4.5
- (ii) Describe security market line. 2
- (a) (i) Define and describe minimum-variance set, efficient frontier of feasible set for any given n assets. 4
- (ii) Define total return of an asset. What is beta of a portfolio ? 2.5
- (b) Consider two 5-year bonds: one has a 9% coupon and sells for 101.00; the other has a 7% coupon and sells for 93.20. Find the price of a 5-year zero-coupon bond. Both bonds have the same face value normalized to 100. 6.5

- (c) (i) Let the risk-free rate be $r_f =$ Suppose the rate of return of the market has an expected value of 12% and a standard deviation of 15%. Consider an asset having covariance 0.045 with the market. Find β and the expected rate of return of asset.
- (ii) Assume that the expected rate of return on the market portfolio is 23% and the rate of return on T-bills (the risk-free rate) is 7%. The standard deviation of the market is 32%. Assume that the market portfolio is efficient. What is the equation of the capital market line?
5. (a) (i) Explain what is a short call position and a long put position in an American option.
- (ii) Give differences between forward and futures contracts. Illustrate with examples.

(b) Consider a long forward contract to purchase a non-dividend paying stock in 3 months. Assume the current price is \$35, the 3-months risk-free interest rate r is 5% per annum, forward price is \$38. Is there a possibility of arbitrage? Explain. 6

(c) An investor sells a European call option on a share for \$4. The stock price is \$47 and the strike price is \$50. Under what circumstances does the investor make a profit? Under what circumstances will the option be exercised? Draw a diagram showing the variation of the investor's profit with the stock price at the maturity of the option. 6

6. (a) Suppose price of a stock is \$31, exercise price is \$30, risk-free interest rate is 10% per annum, the price of a 3 month European call option is \$3 and the price of a 3 month European put option is \$1. Is there put-call parity? Can an arbitrageur make profit at the end of 3 months? Explain. 6.5

- (b) (i) Draw and explain profit from buying a European put option on one share of stock, given option price is \$7 and strike price is \$70.
- (ii) List six factors that affect stock option prices.
- (c) (i) Give three reasons why the treasurer of a company might not hedge the company's exposure to a particular risk.
- (ii) Explain the difference between hedging and arbitrage. Give an example for each.

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